End-of-pipe emissions abatement technologies in a CGE-model

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August 25, 2020

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Resumé

Firms may have the opportunity to reduce a share of their greenhouse gas emissions by using abatement technologies, that do not otherwise impact the production process. This type of »end-of-pipe« abatement technologies is typically characterized by a unit cost per abated unit of emissions as well as an abatement potential. This paper describes how a catalogue of such technologies can be encorporated into a computable general equilibrium model (CGE-model). Technology catalogues typically characerize the technologies as having constant marginal costs. This implies that a given technology is either implemented by all firms or not at all. CGE-models that are solved using constrained nonlinear optimization are not able to account for corner cases like this. However, by introducing heterogeneity between firms, the technology catalogue is able to be encorporated. This heterogeneity is used to smooth the problem, which makes it possible to solve for the firms' choices analytically. This paper outlines how this can be done. In practice, the degree heterogeneity can be made arbitrarily small. Thus, constant marginal costs are an edge case of the described solution.

This is an update of an earlier memo with the same title. Since the initial memo from February, a new section 3 with extensions to the framework has been added.

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1 Theory

Consider a representative firm which produces emissions of quantitye^0 by using input x:

$$e^0 = \eta x \tag{1}$$

where η is the emission coefficient. This is the quantity emitted if the firm does nothing to abate its emissions.

Assume that there exists T emissions-reducing technologies (a technology catalogue). Technology t is characterized by a unit cost, c_t, and a reduction share, q_t, which represents the share of emissions, which the technology can abate. The combined emissions reduction from using the technology is therefore given by $\Delta e_t = q_t e^0$.

Assume now that an emissions tax, τ, is implemented. A technology defined by (q_t, c_t) will provide the firm with savings $\tau q_t e^0$ and costs $c_t q_t e^0$. It is therefore a necessary condition for the technology t to be used, that

$$\tau \geq c_t. \tag{2}$$

If (2) is satisfied, a rational firm will install the emissions-reducing technology t. We now introduce an element of heterogeneity, such that the costs for a given firm can by higher or lower than c_t, but the average firm faces the cost c_t. We assume that firm j introduces technology t if:

$$\tau \geq c_{jt}$$

where c_{jt} is firm j's cost from using the technology. We now assume that there are many firms and that c_{jt} is a log-normally distributed variable:

$$log\left(c_{jt}\right) \sim N\left(log\left(c_t\right) - \frac{\sigma^2}{2}, \sigma^2\right) \tag{3}$$

where c_t is the average unit cost for technology t according to the tehcnology catalogue. The specification (3) ensures that all firms have positive costs from using the technology and that

$$E\left[c_{jt}\right] = c_t \tag{4}$$

Let $\phi(x)$ be the probability density function and $\Phi(x)$ the cumulative distribution function for the standardized normal distribution. The share of firms that introduce technology t is then given by:

$$I_t = \Phi\left(\frac{log\left(\tau\right) - log\left(c_t\right) + \frac{\sigma^2}{2}}{\sigma}\right) \tag{5}$$

For a given tax τ, the firms' total reduction of emissions is given by

$$E = e^0 \sum_{t=1}^{T} q_t I_t \tag{6}$$

It is slightly harder to calculate the firms' total costs, since this requires us to consider the effect of firm-heterogeneity. Let C_t be the firms' total expenditure on technology t. It is then the case that the costs for a given technology are given by:

$$\begin{aligned}
C_t &= e^0 \int_0^\tau q_t c \phi\left(\frac{log\left(c\right) - log\left(c_t\right) + \frac{\sigma^2}{2}}{\sigma}\right) dc \\
&= \Phi\left(\frac{log\left(\tau\right) - log\left(c_t\right) + \frac{\sigma^2}{2}}{\sigma}\right) e^0 q_t \int_0^\tau c \frac{\phi\left(\frac{log(c) - log(c_t) + \frac{\sigma^2}{2}}{c}\right)}{\Phi\left(\frac{log(\tau) - log(c_t) + \frac{\sigma^2}{2}}{\sigma}\right)} dc \\
&= \Phi\left(\frac{log\left(\tau\right) - log\left(c_t\right) + \frac{\sigma^2}{2}}{\sigma}\right) e^0 q_t E\left[c | c \leq \tau\right] \\
&= \Phi\left(\frac{log\left(\tau\right) - log\left(c_t\right) + \frac{\sigma^2}{2}}{\sigma}\right) e^0 q_t c_t \frac{\Phi\left(\frac{log(\tau) - log(c_t) + \frac{\sigma^2}{2} - \sigma^2}{\sigma}\right)}{\Phi\left(\frac{log(\tau) - log(c_t) + \frac{\sigma^2}{2}}{\sigma}\right)} \\
&= e^0 q_t c_t \Phi\left(\frac{log\left(\tau\right) - log\left(c_t\right) - \frac{\sigma^2}{2}}{\sigma}\right)
\end{aligned}$$

We have used the fact that $E\left[x | x \leq a\right] = E[x] \frac{\Phi(\sigma - (\ln(a) - \mu)/\sigma}{\Phi(\ln(a) - \mu)/\sigma)}$ for $log(x) \sim N\left(log\left(\mu\right), \sigma^2\right)$[1]. This implies that the total costs of using technologies from the technology catalogue are:

$$C = e^0 \sum_{t=1}^{T} q_t c_t I_t^c \tag{7}$$

where

$$I_t^c = \Phi\left(\frac{log\left(\tau\right) - log\left(c_t\right) - \frac{\sigma^2}{2}}{\sigma}\right) \tag{8}$$

If all firms used technology t, the costs from the technology would be given by $e^0 q_t c_t$. I_t^c represents the share of the total potential costs from technology t, which are realized by the firms.

The parameter σ represents the degree of heterogeneity between firms. One possibility is to choose a low σ, which in practice means that the firms have almost identical costs. This implies that the share functions I_t and I_t^c approach 0 or 1, suggesting that either all or none of the firms choose to use the technology.

[1] Se e.g. http://home.datacomm.ch/paulsoderlind/Courses/OldCourses/EcmXSta.pdf p. 17.

2 Application in a CGE-model

From (1) and (7), the total abatement costs are given by

$$C = \eta\zeta_c(\tau)\,x \tag{9}$$

where

$$\zeta_c(\tau) \equiv \sum_{t=1}^{T} q_t c_t I_t^c$$

and from (1) and (6), the total emissions are given by

$$E = \eta\left(1 - \zeta_q(\tau)\right)x \tag{10}$$

where

$$\zeta_q(\tau) \equiv \sum_{t=1}^{T} q_t I_t$$

The total costs related to input x are:

$$\hat{p}_x = \left(p_x + \eta\zeta(\tau)\right)x$$

where

$$\zeta(\tau) \equiv \zeta_c(\tau) + \tau\left(1 - \zeta_q(\tau)\right) \tag{11}$$

given that abatement costs and taxes on emissions are included.

This implies that

$$\zeta(\tau) = \tau - \sum_{t=1}^{T} q_t\left[\tau I_t - c_t I_t^c\right]$$

Assume now that the firm's demand for good x enters the CES-nest with another good z and that these two goods form an aggregate good H. This is a standard CES-nest:

$$x = \mu_x\left(\frac{p_x + \eta\zeta(\tau)}{P_H}\right)^{-E} H \tag{12}$$

$$z = \mu_z\left(\frac{p_z}{P_H}\right)^{-E} H \tag{13}$$

$$P_H H = \left(p_x + \eta\zeta(\tau)\right)x + p_z z \tag{14}$$

where E is the elasticity of substitution between x and z. The CES price index P_H is implicitly derived from (12)-(14). It can be shown that this price index is given by:

$$P_H = \left[\mu_x\left(p_x + \eta\zeta(\tau)\right)^{1-E} + \mu_z p_z^{1-E}\right]^{\frac{1}{1-E}} \tag{15}$$

It is nessecary to include either (14) or (15) in the model.

3 Extensions

In this section, we discuss a few extensions to the framework presented above.

3.1 One technology affects multiple emissions

We may have technologies that affect multiple types of emissions that are all proportional to use of the same input. This is for instance the case in agriculture, where some technologies can reduce emissions of both ammonia and methane from manure. In this case, we can extend the framework by adding an index, ϵ, to denote the type of emissions. For instance, emissions by different emissions types are given by:

$$e_\epsilon^0 = \eta_\epsilon x \tag{16}$$

The technology will once again be implemented if savings from reduced tax payments is higher than the cost of the technology. The total potential savings are therefore $x \sum_\epsilon \tau_\epsilon q_{t,\epsilon} \eta_\epsilon$. We redefine the unit costs of the technology to be the cost per unit of the polluting input and denote this $\tilde{c}$. This means that the firm will use the technology if:

$$x \sum_\epsilon \tau_\epsilon \eta_\epsilon q_{t,\epsilon} \geq \tilde{c}_t x \;\Rightarrow\; \tilde{\tau} \equiv \sum_\epsilon \tau_\epsilon \eta_\epsilon q_{t,\epsilon} \geq \tilde{c}_t \tag{17}$$

where the potential savings per unit of the polluting input is $\tilde{\tau}$. The rest of the equations of the framework follows from this. Below, the key equations are reproduced, where tildes indicate that expressions are modified due to multiple emissions sources and the change in definition of unit costs. Total abatment costs are given by

$$\tilde{C} = \tilde{\zeta}_c \left(\tilde{\tau}\right) x \tag{18}$$

where

$$\tilde{\zeta}_c \left(\tilde{\tau}\right) \equiv \sum_{t=1}^{T} \tilde{c}_t I_t^c(\tilde{\tau}, \tilde{c}_t)$$

Total emissions of type ϵ are given by

$$E_\epsilon = \eta_\epsilon \left(1 - \tilde{\zeta}_{q,\epsilon}\left(\tilde{\tau}\right)\right) x \tag{19}$$

where

$$\zeta_{q,\epsilon} \left(\tilde{\tau}\right) \equiv \sum_{t=1}^{T} \tilde{q}_{t,\epsilon} I_t(\tilde{\tau}, \tilde{c}_t)$$

The total costs related to input x are:

$$\hat{p}_x = \left(p_x + \tilde{\zeta}\left(\tilde{\tau}\right)\right) x$$

where

$$\tilde{\zeta}(\tilde{\tau}) \equiv \tilde{\zeta}_c(\tilde{\tau}) + \sum_{\epsilon} \tau_{\epsilon} \left(1 - \tilde{\zeta}_{q,\epsilon}(\tilde{\tau})\right) \tag{20}$$

Firm demands and budget constraints are given by:

$$x = \mu_x \left(\frac{p_x + \eta \tilde{\zeta}(\tilde{\tau})}{P_H} \right)^{-E} H \tag{21}$$

$$z = \mu_z \left(\frac{p_z}{P_H} \right)^{-E} H \tag{22}$$

$$P_H H = \left(p_x + \eta \tilde{\zeta}(\tilde{\tau})\right) x + p_z z \tag{23}$$

3.2 Introducing a shadow tax

It is useful to be able to model technology adoptation in the absence of an emissions tax. This may be relevant during model calibration, where we wish to calibrate to some fixed level of takeup of the technology, even though taxes do not change. It may also be for model simulations where command-and-control-type regulations force technology adoptation, without the firms paying emissions taxes. For these situations, we can introduce a »shadow tax«, which enters into the expression for the share of firms that adopt the technology, 5, but not into the expression for firm costs, 8, since the tax is not actually paid. The sum of regular taxes and the shadow tax is an expression of the costs related to technology adoptation for the marginal adopter.

The shadow tax is by default exogenous and equal to zero, but may be endogenized to hit a certain adoptation share, either during model calibration or as part of a simulation.

4 Concluding remarks

This paper describes a method for how end-of-pipe abatement technologies can be integrated in a CGE-model. The idea is to use information from the technology catalogues as directly as possible in the CGE-model. To the best of our knowledge, this method described in this paper uses information from a technology catalogue in a more direct fashion than the existing methods for integrating abatement technologies in CGE-models in the literature (for example Kluila and Rutherford (2013) and Weitzel et al (2019)). To integrate the technology catalogue, it is assumed that the firms have heterogenous cost from using different technologies.

In the following, we discuss a number of characteristics of the modelling described in this paper.

Polution abatement occurs without profit

A challenge for modelling polution abatement is that the modelling choice may give rise to profit in the sector. This is the case, since the cost from using a technology can be lower than the cost, which the firm saves from avoiding the tax. The problem can be described in the following general way. Assume that a firm produces a good, H, using two inputs and that there are constant returns to scale in the production funciton $H(x,z)$. The good can be sold at price P_H and the inputs can be purchased at constant prices (p_x, p_z). The value of production then corresponds with the returns to the factors of production, ie. $P_H H(x,z) = P_H \frac{\partial H}{\partial x} x + P_H \frac{\partial H}{\partial y} z$, which corresponds with zero profit in the absence of polution abatement. Assume now that the firm's consumption of input x produces emissions, E. The emissions are subject to the tax rate τ. Some of the emissions can be abated if the firm takes action a: $E = E(x,a)$. This action has the cost $C = C(x,a)$. The firm's profit is then given by:

$$\Pi = P_H H(x,z) - p_x x - p_z z - C(a,x) - \tau E(a,x) \tag{24}$$

The first order conditions for profit maximization are then:

$$\frac{\partial \Pi}{\partial x} = 0 \quad \Rightarrow \quad P_H \frac{\partial H}{\partial x} = p_x + \frac{\partial C}{\partial x} + \tau \frac{\partial E}{\partial x} \tag{25}$$

$$\frac{\partial \Pi}{\partial z} = 0 \quad \Rightarrow \quad P_H \frac{\partial H}{\partial z} = p_z \tag{26}$$

$$\frac{\partial \Pi}{\partial a} = 0 \quad \Rightarrow \quad \frac{\partial C}{\partial a} = -\tau \frac{\partial e}{\partial a} \tag{27}$$

We rearrange and insert (25) and (26) in (24):

$$\begin{aligned}
\Pi &= P_H H(x,y) - (P_H \frac{\partial H}{\partial x} - \frac{\partial C}{\partial x} - \tau \frac{\partial E}{\partial x})x + P_H \frac{\partial H}{\partial z} z - C(a,x) - \tau e(a,x) \\
&= P_H H(x,y) - P_H \frac{\partial H}{\partial x} x + \frac{\partial C}{\partial x} x + \tau \frac{\partial E}{\partial x} x + P_H \frac{\partial H}{\partial z} z - C(a,x) - \tau e(a,x) \\
&= \frac{\partial C}{\partial x} x + \tau \frac{\partial E}{\partial x} x - C(a,x) - \tau E(a,x) \\
&\gtreqless 0
\end{aligned}$$

where the last equality follows from the assumption of constant returns to scale. As such, profit is not guaranteed to equal zero in the case with polution abatement. In the special case where there is a constant emission coefficient, η, per unit of emission and where the total cost of (technology specific) emissions abatement is proportional to the quantity of emissions, profits will equal zero.[2] This is the case in the model described in this paper.

[2]In this case, $E(a,x) = E(a)\eta x \;\Rightarrow\; \frac{\partial E}{\partial x} x = E(a,x)$ and $\frac{\partial C}{\partial x} x = C(a,x)$.

Using the one-emission-type verison of the framework, this can be seen by inserting (14) in (28) and inserting (9) and (10) in (30):

$$\begin{aligned}
\Pi &= P_H H(x,y) - p_x x - p_z z - C(a,x) - \tau E(a,x) & (28)\\
&= (p_x + \eta\zeta(\tau))\, x + p_z z - p_x x - p_z z - C(a,x) - \tau E(a,x) & (29)\\
&= \eta\zeta(\tau) - C(a,x) - \tau E(a,x) & (30)\\
&= \eta\zeta(\tau) - \eta\zeta_c(\tau)\, x - \tau\eta\,(1 - \zeta_q(\tau))\, x & (31)\\
&= 0 & (32)
\end{aligned}$$

The last equality follows from (11). The intuition behind this is that the profit which could result from using the abatement technology is cancelled out by a change in the output price.

Heterogeneity

Heterogeneity is often a reasonable assumption, since firms in practice are heterogenous. A concrete example of this in the use of nitrification inhibitors, which farmers can put in animal feed to reduce greenhouse gas emissions. It is not unlikely that the unit price for nitrification inhibitors faced by an individual farm can be affected by differentiated prices from the producer, farm-specific volume discounts as well as the timing of the agreement between the producer and the farm. Alternatively, heterogeneity can be interpreted as information-heterogeneity, where there is uncertainty about the actual cost of using a technology, but the firm on average has accurate beliefs about the costs. If $c_{jt} = c_t$, the firm's belief about the unit costs are accurate, while the firm for example overestimates the unit costs if $c_{jt} > c_t$.

Even though there may be good reasons for there to be cost-heterogeneity in practice, it is possible to choose a small enough degree of heterogeneity, that all firms in the model either use or do not use a given technology.

Choice of cost defintion

There may be technologies in the technology catalogue which are profitable given the existing prices and taxes, i.e. $\tau > c_t$ (for example because $\tau = 0$ and $c_t < 0$), but which are not used. If these technologies are implemented directly in a CGE-model, the agents in the model will immediately start using the technology. This gives rise to a discrepancy between the model's predictions and the real world. In such situations, it is necessary to explain why the technologies are not used, when it appears that they would save the firms money.

There are multiple possible explainations for this apparent paradox. The literature often refers to unrealistic discount rates, optimistic estimates of the technology's effectiveness or

»hidden« costs related to using the technology (for example from increased time consumption or complexity of production), c.f. Alcott and Greenstone (2012) among others.

In practice it may be necessary to take these things into consideration when integrating the technology catalogue and the CGE-model. This can for example be done by increasing the costs in the technology catalogue, such that $\tau < c_t$ initially for all t. [3] Another possibility is that the technologies are in fact used in the baseline, which implies that the technology catalogue would overestimate the potential of the technologies. In this case, it would be neccesary to reduce q_t.

References

Allcott, H. and M. Greenstone (2012): Is There an Energy Efficiency Gap? The Journal of Economic Perspectives, 26(1), p. 3–28.

Kiuila, O. and T.F. Rutherford (2013): The cost of reducing CO 2 emissions: Integrating abatement technologies into economic modeling. Ecological Economics 87, p. 62-71

Weitzel, M., B. Saveyn and T. Vandyck (2019): Including bottom-up emission abatement technologies in a large-scale global economic model for policy assessments. Energy Economics 83, p. 254-263.

[3] There are other possible explanations for why firms don't use abatement technologies, e.g., firms can have incomplete information, face capital constraints or be risk averse. If these are the drivers behind the technologies not being used, it will not be entirely correct to increase technology costs in order to force the technologies to not being used in the model baseline.